ECS 452: Exercise 3

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former groups.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Name	ID

1. Consider two random variables *X* and *Y* whose joint pmf matrix is given by

$$\mathbf{P} = \begin{array}{c} x \backslash y \\ 1 \\ 2 \\ q(y) \end{array} \begin{array}{c} \begin{array}{cc} 3 & 5 \end{array} \\ \begin{bmatrix} 1/18 & 5/18 \\ 4/9 & 2/9 \end{bmatrix} \\ \begin{array}{cc} 9/18 & 9/18 \end{array} \end{array} \begin{array}{l} p(x) \\ \xrightarrow{\Sigma} 6/18 = 1/3 \\ \xrightarrow{\Sigma} 6/9 = 2/3 \end{array}$$

Find

a. $H(X,Y) = -\frac{1}{18}\log_2\frac{1}{18} - \frac{5}{18}\log_2\frac{5}{18} - \frac{4}{9}\log_2\frac{4}{9} - \frac{2}{9}\log_2\frac{2}{9} \approx 1.7472$ bits (per symbol)

b. $H(X) = H([1/3 \quad 2/3]) = -\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3} \approx 0.9183$ bits (per symbol)

c. $H(Y) = H([1/2 \quad 1/2]) = \log_2 2 = 1$ bit (per symbol)

↑ For uniform RV, the entropy is simply $\log_2$ of the size of its support.

d. $I(X;Y) = H(X) + H(Y) - H(X,Y) \approx 0.9183 + 1 - 1.7472 \approx 0.1711$ bits

e. **Q** matrix

$$P = \begin{bmatrix} 1/18 & 5/18 \\ 4/9 & 2/9 \end{bmatrix} \begin{array}{l} \times 3 \\ \times 3/2 \end{array} \quad \times \frac{1}{p(x)}$$

$$\begin{array}{c} x \backslash y \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 3 & 5 \end{array} \\ \begin{bmatrix} 1/6 & 5/6 \\ 2/3 & 1/3 \end{bmatrix} \end{array} = Q$$

f. $H(Y|1) = H([1/6 \quad 5/6]) \approx 0.65$ bits (per symbol)

g. $H(Y|2) = H([2/3 \quad 1/3]) \approx 0.9183$ bits (per symbol)

h. $H(Y|X) = H(X,Y) - H(X) \approx 1.7472 - 0.9183 \approx 0.8289$ bits (per symbol)

Alternatively $= p(1)H(Y|1) + p(2)H(Y|2) \approx \frac{1}{3}\times 0.65 + \frac{2}{3}\times 0.9183 \approx 0.8289$